

# Inverse of a function

If  $f : X \rightarrow Y$  such that  $f(x) = y$

is one - one and onto,

then we can define a unique function

$g : Y \rightarrow X$  such that  $g(y) = x, x \in X, y \in Y$ .

$g$  is called inverse of  $f$  and is denoted by  $f^{-1}$ .

# Example

If  $f : R \rightarrow R$  be defined as  $f(x) = x + 1$ ,  
find inverse of  $f$ .

$f$  is one - one and onto

we can define inverse of  $f$ ,

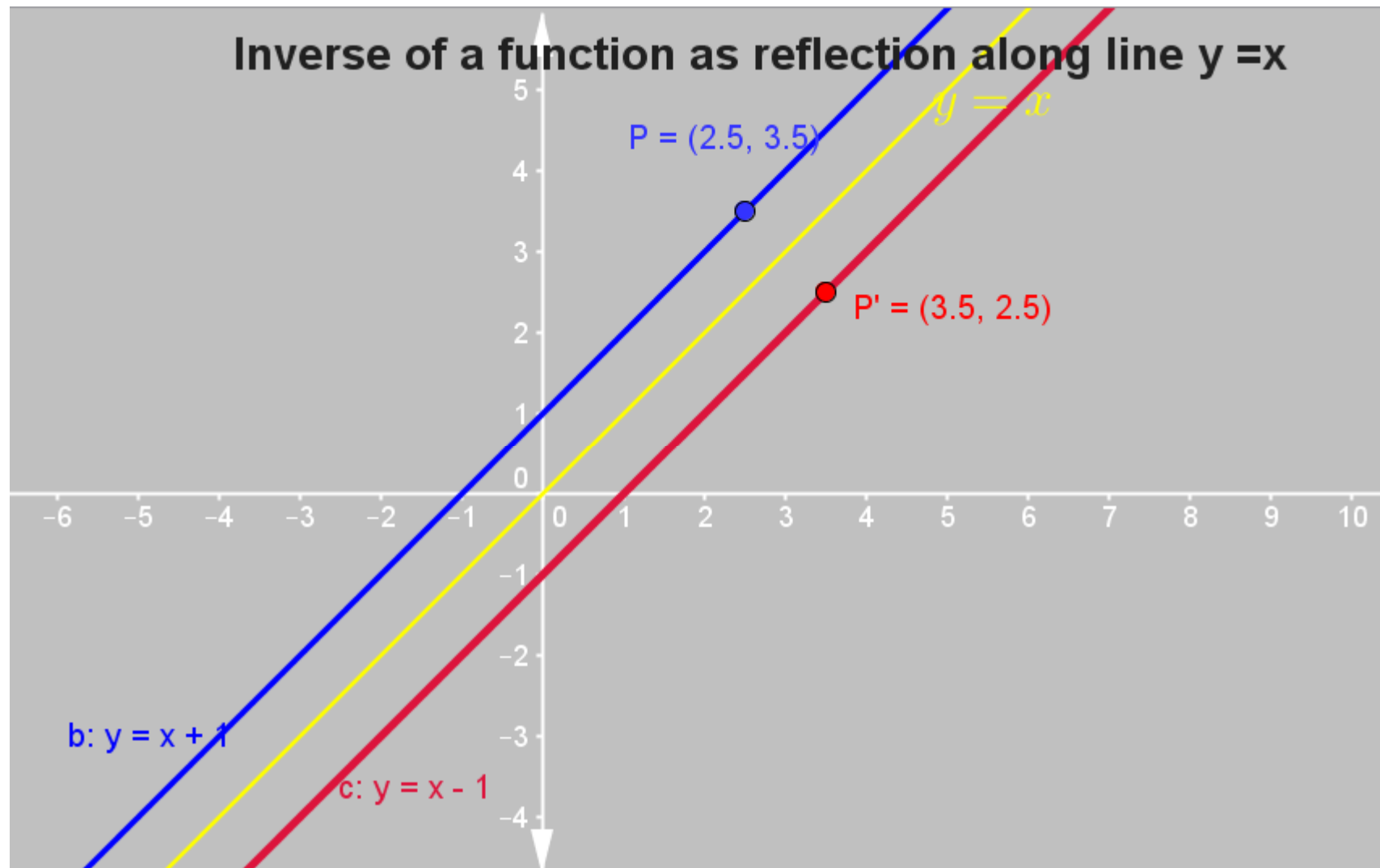
$f^{-1} : R \rightarrow R$  such that  $f^{-1}(y) = y - 1, y \in R$ .

# Graphical interpretation of Inverse of a function

If  $(1,2)$  lies on graph of  $f$ , then  
 $(2,1)$  lies on graph of  $f^{-1}$ .

$f^{-1}$  is reflection of  $f$  along the line  $y = x$ .

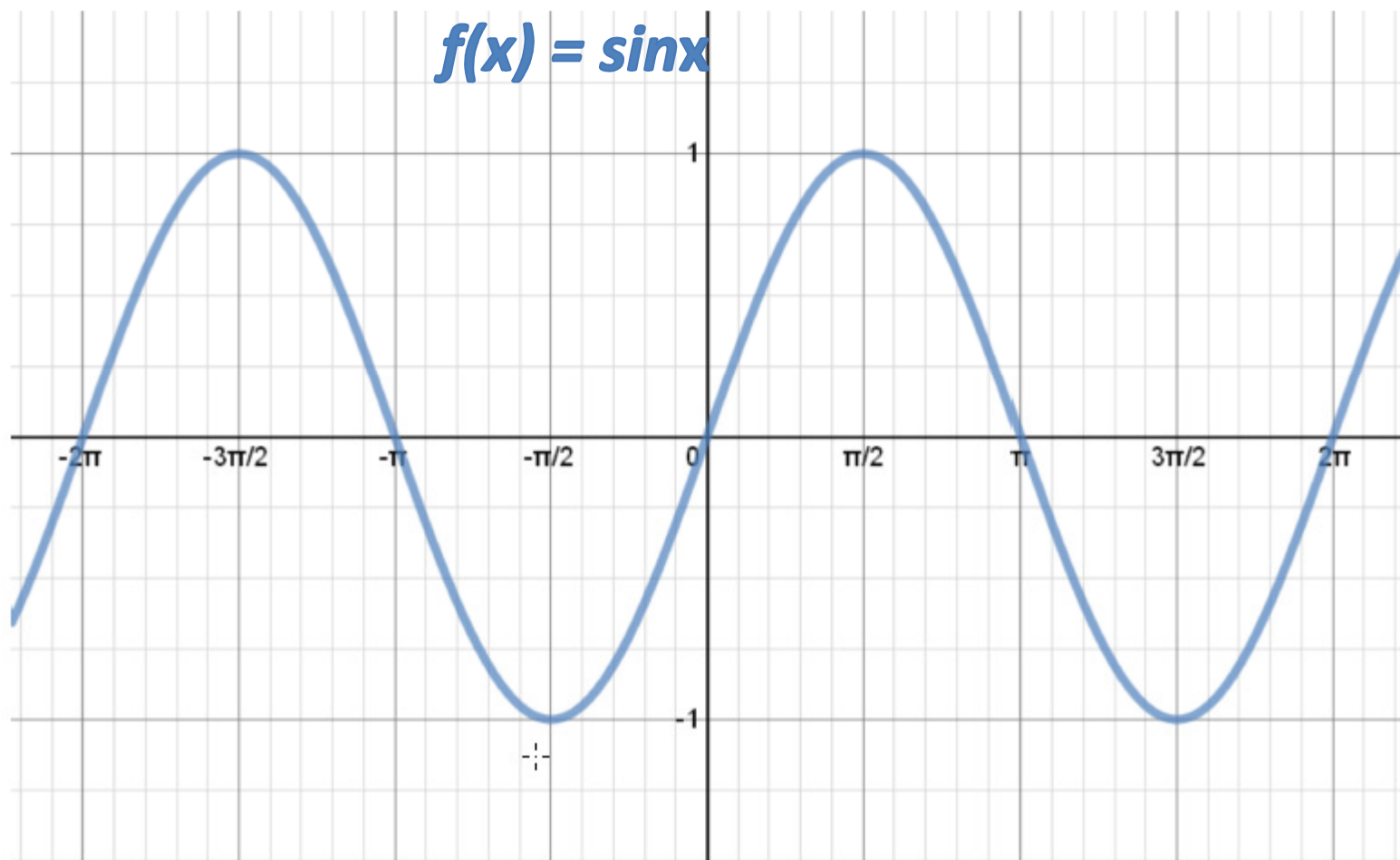
## Inverse of a function as reflection along line $y = x$



Result:

A function  $f$  is invertible if and only if  $f$  is **one-one** and **onto**.

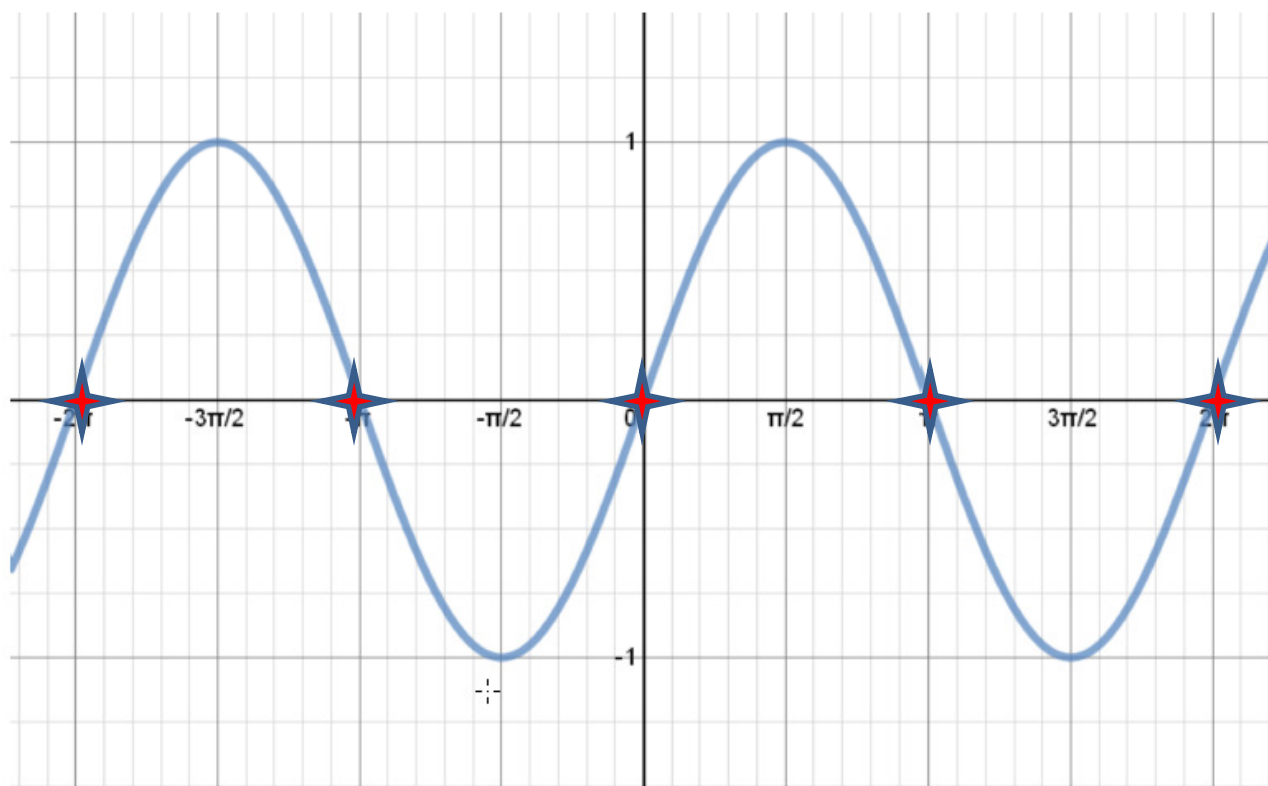
$$f(x) = \sin x$$



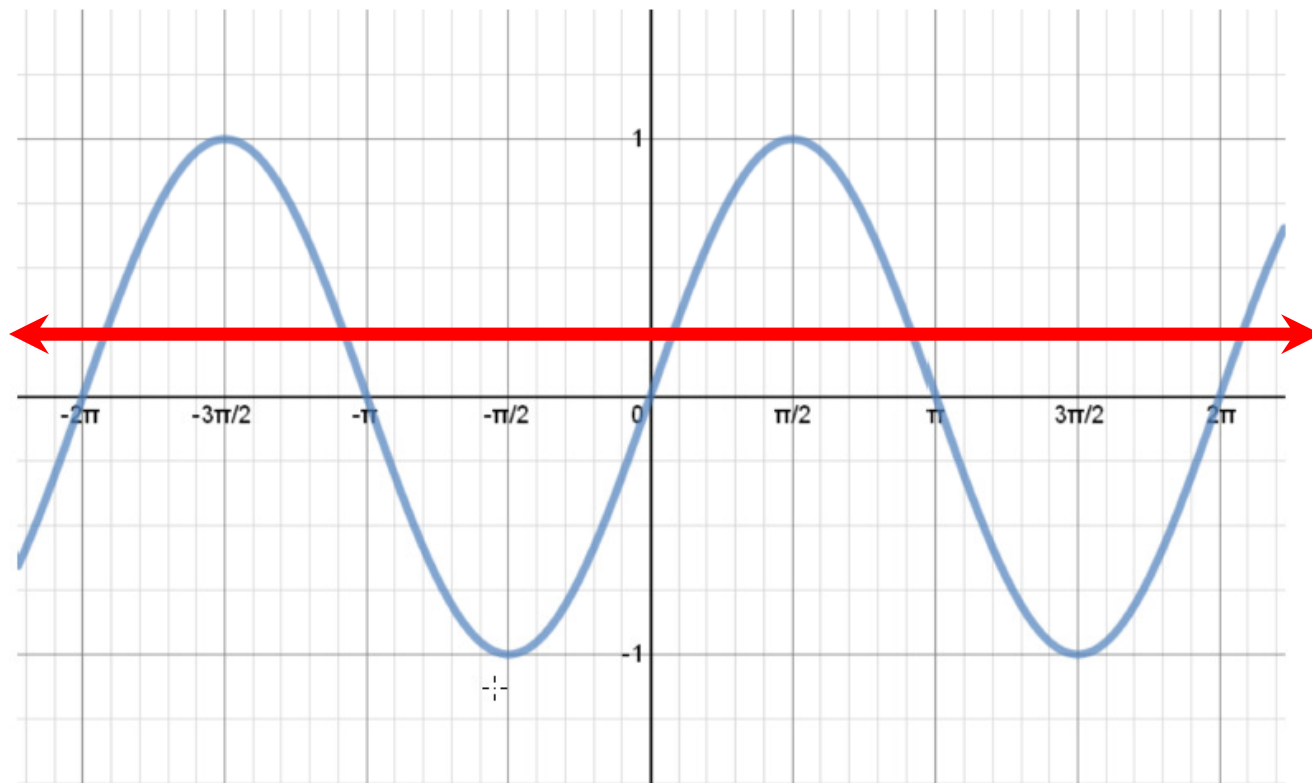
$$f(x) = \sin x$$

Domain of  $f$  = All real numbers,  $\mathbb{R}$

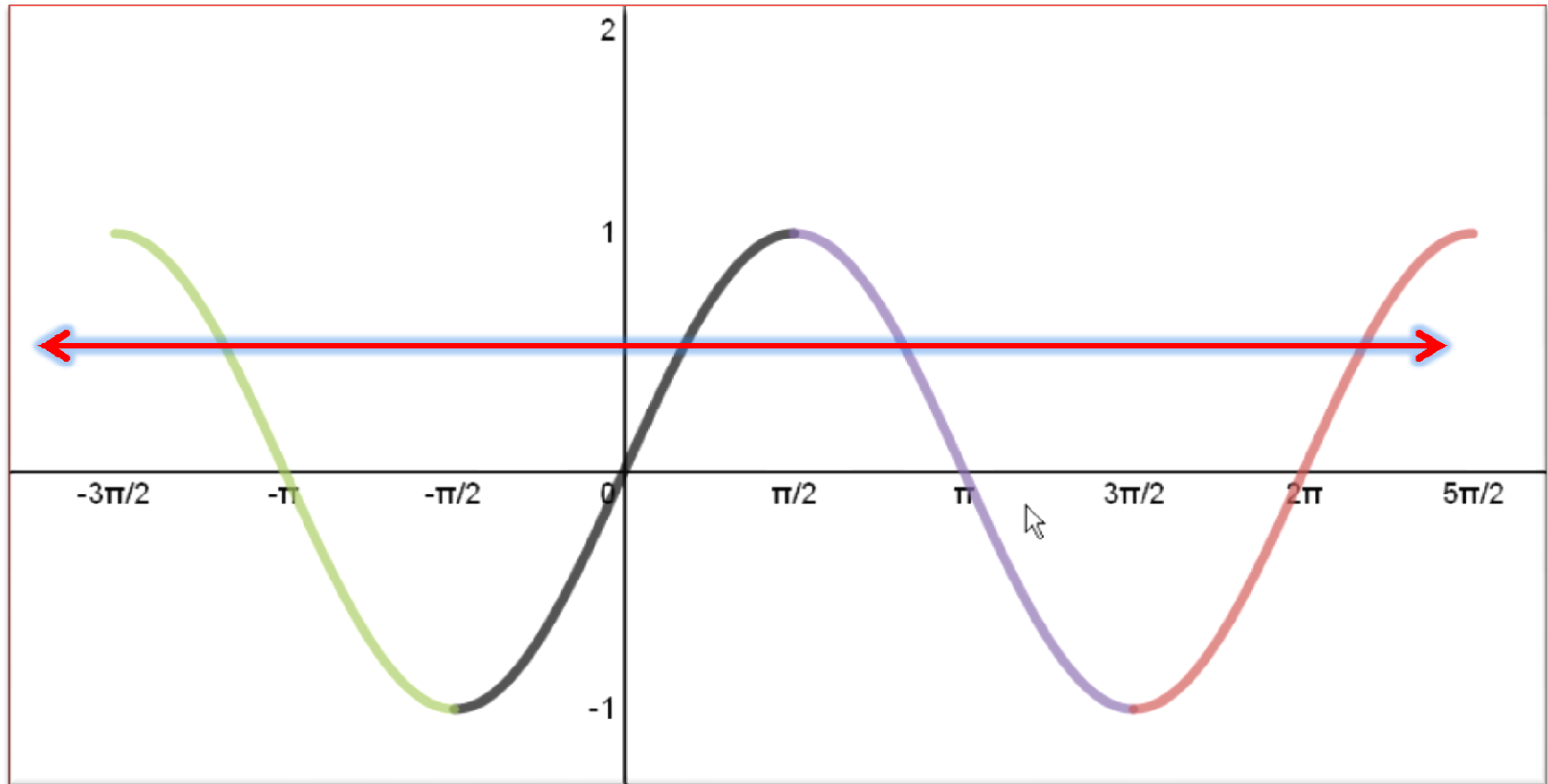
Range of  $f = [-1, 1]$



# Horizontal line test



$$f(x) = \sin(x)$$



$$\text{sine} : \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] \rightarrow [-1, 1]$$

$$\text{sine} : \left[ \frac{\pi}{2}, \frac{3\pi}{2} \right] \rightarrow [-1, 1]$$

$$\text{sine} : \left[ \frac{-3\pi}{2}, \frac{-\pi}{2} \right] \rightarrow [-1, 1]$$

is one - one and onto

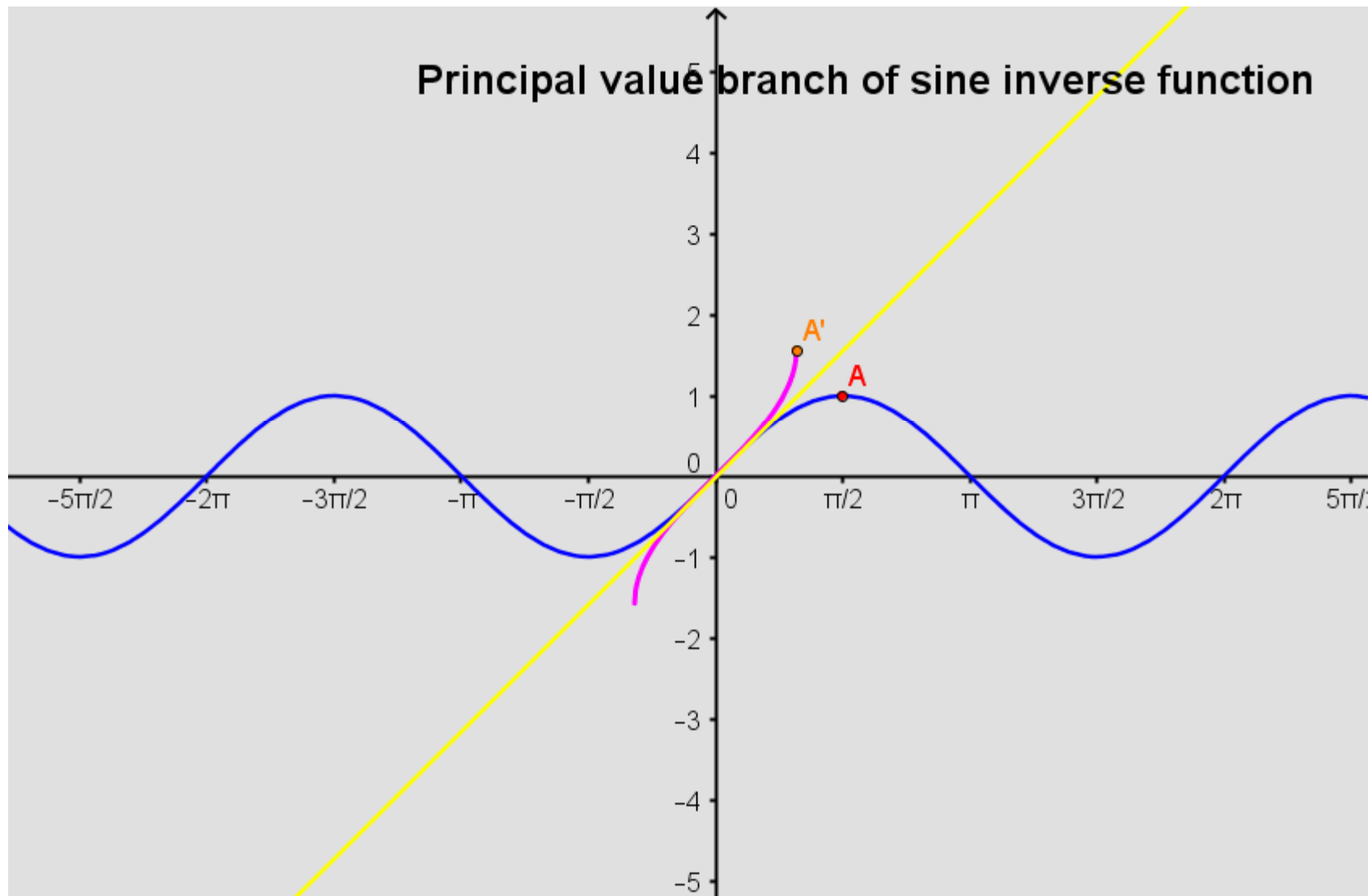
We define,

$$\sin^{-1} : [-1, 1] \rightarrow \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$\sin^{-1}(x) = y \text{ if } x = \sin y$$

This branch is called the principal value branch.

# Principal value branch of sine inverse function



Q : Find the principal value of the following :

$$(a) \sin^{-1}(-1)$$

$$(b) \sin^{-1}\left(\frac{1}{2}\right)$$

Solution :

$$(a) \sin^{-1}(-1)$$

$$\text{Let } y = \sin^{-1}(-1)$$

$$\Rightarrow \sin y = -1$$

$$\text{For principal value } y \in \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right]$$

$$\therefore y = \frac{-\pi}{2}$$

$$(b) \sin^{-1}\left(\frac{1}{2}\right)$$

$$\text{Let } y = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \sin y = \frac{1}{2}$$

$$\text{For principal value } y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore y = \frac{\pi}{6}$$